

Digging deeper: Observing primordial gravitational waves below black hole binary confusion noise

T. Regimbau,^{1, a} M. Evans,² N. Christensen,^{1, 3, b} E. Katsavounidis,² B. Sathyaprakash,^{4, 5, c} and S. Vitale²

¹*Artemis, Université Côte d'Azur, CNRS, Observatoire Côte d'Azur, CS 34229, Nice cedex 4, France*

²*LIGO, Massachusetts Institute of Technology, Cambridge, MA 02139, USA*

³*Physics and Astronomy, Carleton College, Northfield MN 55057, USA*

⁴*Department of Physics, The Pennsylvania State University, University Park, PA 16802, USA*

⁵*School of Physics and Astronomy, Cardiff University, Cardiff, CF24 3AA, UK*

(Dated: December 6, 2016)

The merger rate of black hole binaries inferred from the recent LIGO detections implies that a stochastic background produced by a cosmological population of mergers will likely mask the primordial gravitational-wave background. Here we demonstrate that the next generation of ground-based detectors, such as the Einstein Telescope and Cosmic Explorer, will be able to observe binary black hole mergers throughout the universe with sufficient efficiency that the confusion background can be subtracted to observe the primordial background at the level of $\Omega_{\text{GW}} \simeq 10^{-13}$ after five years of observation.

PACS numbers: 04.80.Nn, 04.25.dg, 95.85.Sz, 97.80.-d

Introduction — According to various cosmological scenarios, we are bathed in a stochastic primordial gravitational-wave background (PGWB) produced in the very early stages of the Universe. Proposed theoretical models include the amplification of vacuum fluctuations during inflation [1–3], pre-Big Bang models [4–6], cosmic (super)strings [7–10] or phase transitions [11–13]. The detection of a primordial background would have a profound impact on our understanding of the evolution of the Universe, as it represents a unique window on the first instant, up to the limits of the Planck era, and on the physical laws that apply at the highest energy scales.

In addition to the PGWB, an astrophysical background is expected to result from the superposition of a large number of unresolved sources since the beginning of stellar activity (see [14], for a review of different sources that could produce an astrophysical background). The astrophysical background potentially contains a wealth of information about the history and evolution of a population of point sources, but it is a *confusion noise background* that is detrimental to the observation of the PGWB. In this Letter we show that at the sensitivity levels envisaged for third generation detectors such as the Einstein Telescope (ET) and Cosmic Explorer (CE), it is possible to subtract the confusion background from the data enabling the study of the PGWB. This problem is similar to the one investigated by [15, 16] in the context of the Big Bang Observer.

On September 14th, 2015, Advanced LIGO [17–19] directly detected gravitational-waves (GW) from the collision of two stellar-mass black holes at a redshift of $z \sim 0.1$

(GW150914) [20, 21]. The inferred component masses of $m_1 \sim 36 M_\odot$ and $m_2 \sim 29 M_\odot$ are larger than those of candidate black holes in X-ray binaries inferred from reliable dynamical measurements [22]. LIGO detections suggest the existence of a population of black holes with relatively large masses, that might have formed in low-metallicity stellar environments [22], either through the evolution of an isolated massive binary in a galaxy [23] or through mass segregation and dynamical interactions in a dense globular system [24].

LIGO discoveries during the first observing run included a high-confidence ($> 5\sigma$) detection of a second merger event GW151226 and a marginal event of lower significance ($< 2\sigma$) LVT151012, both believed to be binary black hole (BBH) mergers. GW151226 resulted from the merger of black holes of mass $m_1 = 14.2 M_\odot$ and $m_2 = 7.5 M_\odot$ [25], and LVT151012 is believed to have resulted from the merger of black holes of mass $m_1 = 23 M_\odot$ and $m_2 = 13 M_\odot$. These observations indicate that many more detections would occur in the future and have provided the tightest constraints on the rate of such events [26].

Besides the loudest and closest events that can be detected individually, the population of undetected sources at larger redshift is expected to create a significant astrophysical background. The background from the population of compact binaries has been investigated by many authors in the past (see [14, 27–32], for the most recent papers), who suggested that Advanced LIGO and Advanced Virgo had a realistic chance of detecting this background after a few years of operation with the standard cross-correlation method, even if this background is not continuous (no overlap of the sources) or Gaussian [33].

In Ref. [34] the LIGO and Virgo collaborations calculated the contribution to the stochastic background from

^a regimbau@oca.eu

^b nelson.christensen@oca.eu

^c bss25@psu.edu

BBHs with the same masses as GW150914. Taking into account the statistical uncertainty in the rate, they found that the signal could be detected, in the most optimistic case, even before the design sensitivity of the instruments is reached, but more likely after a few years of their operation at design sensitivity. It was also shown that lower mass systems that are too faint to be detected individually could add a significant contribution to the background. Following this first paper, other authors have investigated the implication of GW150914 for the confusion background, including models of metallicity evolution with redshift and mass distributions [35, 36], and arrived at the same conclusion: the background from BBHs is likely to be higher than previously expected and may dominate over the primordial background.

In this paper, we use Monte Carlo simulations to calculate the confusion background from BBHs observed by a network of ground-based detectors. We study the potential reduction in the level of this background as more BBH signals are detected, and can be subtracted from the data, because of the improved sensitivity of ET [37] and CE [38] compared to advanced detectors. We show that the confusion background of astrophysically produced GWs can be significantly reduced, paving the way to observe the primordial background. We do not investigate subtraction data analysis techniques in detail, assuming that the times containing these events can be removed from the search for a residual stochastic gravitational wave background.

Simulation of a population — In order to calculate the total contribution of BBHs to the confusion background, we consider the fiducial model of Ref. [34] and generate an extra-galactic population of BBHs using the Monte Carlo procedure described in [33, 39, 40] and summarized below.

- The masses m_1, m_2 are selected from one of the two astrophysical distributions considered in Ref. [25]: (i) model A: uniform distribution in the logarithm of the component masses $p(m_1, m_2) \propto m_1^{-1} m_2^{-1}$, and (ii) model B: power-law distribution of the primary (i.e., larger mass) companion $p(m_1) \propto m_1^{-2.35}$, and uniform distribution of the secondary. In addition, we require that the component masses take values in the range 5–100 M_\odot with $m_1 + m_2 < 100 M_\odot$.
- The redshift is drawn from a probability distribution $p(z)$

$$p(z) = \frac{R_z(z)}{\int_0^{20} R_z(z) dz} \quad (1)$$

obtained by normalizing the merger rate (in the observer frame) per interval of redshift, over the range $z \in 0-20$, and

$$R_z(z) = \int \frac{R_m(z)}{1+z} \frac{dV}{dz}(z). \quad (2)$$

Here $\frac{dV}{dz}$ is the comoving volume element and R_m (in the source frame) is the rate per volume, given by:

$$R_m(z) = \int_{t_{\min}}^{t_{\max}} R_f(z_f) P(t_d) dt_d, \quad (3)$$

where $R_f(z)$ is the massive binary formation rate, $P(t_d)$ the distribution of the time delay t_d between the formation of the massive progenitors and their merger, z_f is the redshift at the formation time $t_f = t(z) - t_d$, and $t(z)$ is the age of the Universe at merger. The value of R_m at $z = 0$ corresponds to the local rate estimated from the first LIGO observation run [25], which is $30_{-21}^{+43} \text{ Gpc}^{-3} \text{ yr}^{-1}$ for the flat mass distribution and $99_{-70}^{+138} \text{ Gpc}^{-3} \text{ yr}^{-1}$ for the exponential mass distribution.

We assume that R_f follows the cosmic star formation rate and we use the recent model of [41], based on the gamma-ray burst rate of [42] and on the normalization described in [43, 44]. We also assume that black holes of 30 M_\odot or larger can only be formed below the metallicity threshold $Z_c = Z_\odot/2$ [22, 34]. The metallicity is drawn from a \log_{10} -normal distribution with a standard deviation of 0.5 around the mean at each redshift [45] calculated from the mean metallicity-redshift relation of Ref. [46], rescaled upwards by a factor of 3 to account for local observations [41, 47]. We further assume that the time delay distribution follows $P(t_d) \propto 1/t_d$, for a minimal delay of 50 Myr and a maximum time delay equal to the Hubble time.

- The location in the sky $\hat{\Omega}$, the cosine of the orientation ι , the polarization ψ and the phase of the signal at coalescence ϕ_0 , were drawn from uniform distributions.
- For each BBH, we determine if its resultant GW emission is detectable in a given detector network. The signal-to-noise ratio (SNR) detected by matched filtering with an optimum filter in the ideal case of Gaussian noise, in a detector labelled A , is:

$$\rho_A^2 = 4 \int_0^\infty \frac{|F_{+,A} \tilde{h}_+ + F_{\times,A} \tilde{h}_\times|^2}{S_{n,A}} df, \quad (4)$$

where f is the gravitational-wave frequency in the observer frame, \tilde{h}_+ and \tilde{h}_\times the Fourier transforms of the GW strain amplitudes of $+$ and \times polarisations that includes inspiral, merger and ringdown phases of the signal [48], $F_{+,A}$ and $F_{\times,A}$ are the antenna response functions to the GW $+$ and \times polarisations, and $S_{n,A}(f)$ is the one-sided noise power spectral density (PSD) of detector A . The coherent

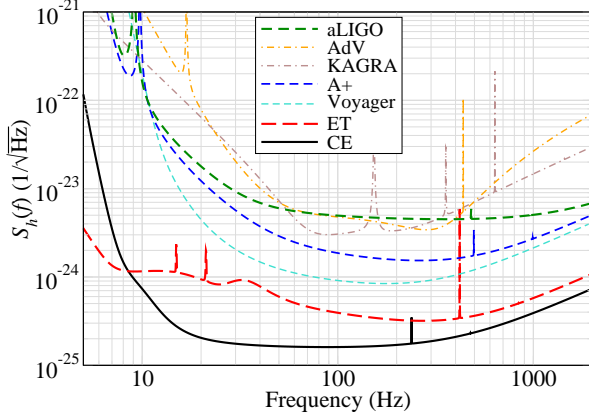


FIG. 1. Design power spectral density of second generation detectors: Advanced LIGO (aLIGO), Advanced Virgo (AdV) and KAGRA and proposed sensitivity of third generation detectors Einstein Telescope (ET) and Cosmic Explorer or (CE). Expected intermediate sensitivities such as Advanced LIGO Plus (A+) and Voyager are also shown.

SNR for a network, assuming uncorrelated noises in the detectors, is simply given by the quadrature sum of the individual SNRs $\rho_T^2 = \sum \rho_A^2$. We assume that only sources with $\rho_T < 12$ contribute to the confusion background.

Binary background — The superposition of the gravi-

Our waveform model includes inspiral, merger and ringdown phases of the signal. In the Newtonian regime, before the black holes reach the last stable orbit, the slope of the spectrum has the well-known $f^{2/3}$ behavior:

$$\Omega_{\text{GW}}(f) = \frac{5\pi^{2/3}G^{5/3}c^{5/3}}{18c^3H_0^2} f^{2/3} \sum_{k=1}^N \frac{(1+z_k)^{5/3}(\mathcal{M}_k)^{5/3}}{D_L(z_k)^2} \left[\frac{(1+\cos^2 \iota_k)^2}{4} + \cos^2 \iota_k \right] \quad (8)$$

where $M = m_1 + m_2$ is the total mass, $\mathcal{M} = (m_1 m_2)^{3/5} M^{-1/5}$ the chirp mass and $D_L(z)$ is the luminosity distance at redshift z . We shall see below that we retrieve this behavior over the relevant range of frequencies.

Results — In this section we investigate the evolution of the background as the sensitivity increases from second to third generation and the number of detectors in the network increases from three to five. The Advanced version of the two LIGO detectors at Hanford (H) and Livingston (L) [18, 19] started collecting data in September 2015 and are expected to reach design sensitivity in 2019, followed by Advanced Virgo (V) a few months later [51]. Two other detectors will join the network over the next eight years: a new detector in India (I) [52] whose sensitivity will be similar to the two LIGO detectors, and the Japanese detector KAGRA (K) [53]. Third generation detectors are currently under design study, such as the Einstein Telescope (ET) [37], which is expected to re-

place Virgo, and the Cosmic Explorer (CE), an upgrade of the LIGO detectors [38]. Between the second and the third generation we expect to reach intermediate sensitivities referred to as A+ and Voyager. Figure 1 plots the strain sensitivity of the various detectors considered in this paper.

$$\Omega_{\text{GW}}(f) = \frac{f}{\rho_c} \frac{d\rho_{\text{GW}}}{df}, \quad (5)$$

where $d\rho_{\text{GW}}$ is the energy density in the frequency interval f to $f + df$, $\rho_c = 3H_0^2 c^2 / 8\pi G$ is the closure energy density of the Universe, and $H_0 = 67.8 \pm 0.9$ km/s/Mpc is the Hubble constant [50].

The GW spectrum from the population of BBHs is given by the expression:

$$\Omega_{\text{GW}}(f) = \frac{1}{\rho_c c} f F(f). \quad (6)$$

where $F(f)$ is the total flux and f is the observed frequency. The total flux (in erg Hz⁻¹) is the sum of the individual contributions:

$$F(f) = T^{-1} \frac{\pi c^3}{2G} f^2 \sum_{k=1}^N (\tilde{h}_{+,k}^2(f) + \tilde{h}_{\times,k}^2(f)) \quad (7)$$

where N is the number of undetected sources in the Monte Carlo sample. The normalization factor T^{-1} assures that the flux has the correct dimension, $T = 1$ yr being the length of the data sample.

place Virgo, and the Cosmic Explorer (CE), an upgrade of the LIGO detectors [38]. Between the second and the third generation we expect to reach intermediate sensitivities referred to as A+ and Voyager. Figure 1 plots the strain sensitivity of the various detectors considered in this paper.

Fig. 2 shows the energy density Ω_{GW} in gravitational waves from unresolved BBHs in Advanced (top plot), A+ (middle plot) and third generation (bottom plot) detectors. Solid (green) curves are the backgrounds for models A (thick lines) and B (thin lines), respectively, when detected BBH signals are not removed from the data, so they are the same in each plot. For each generation of sensitivity, we consider two different networks: A net-

work of 3 detectors (HLV) located at the sites of LIGO-Hanford, LIGO-Livingston and Virgo and a network of 5 detectors (HLVIK) that includes LIGO India and KAGRA, in addition to HLV. In the top plot, the detectors are assumed to have projected sensitivity levels of advanced detectors shown in Fig. 1. In the middle plot, we assume that all the detectors have the same intermediate sensitivity (A+). In the bottom plot, for the third generation we assume the sensitivity of ET in a triangle detector configuration at the location of Virgo and CE for all other detectors.

The BBH confusion background is compared to the minimal detectable flat spectrum $\Omega_{\text{GW}}(f) = \Omega_{\text{min}}$, expected to mimic most of the cosmological backgrounds, and whose value is derived requiring a signal-to-noise ratio of $\rho = 3$ for an observation time $T = 5$ yr, where

$$\rho = \frac{3H_0^2}{10\pi^2} \sqrt{2T} \left[\int_0^\infty df \sum_{i=1}^n \sum_{j>i} \frac{\gamma_{ij}^2(f) \Omega_{\text{GW}}^2(f)}{f^6 P_i(f) P_j(f)} \right]^{1/2}, \quad (9)$$

for a network of detectors $i = 1, 2, \dots, n$. The contribution to the SNR comes mostly from the closest pair of detectors, namely the LIGO Hanford – LIGO Livingston detector pairs over a frequency interval of 50 Hz. The entire network of detectors is needed to identify the signals, estimate their parameters, and then remove their presence from them from the data. In all cases, we observe that the background can be decreased below Ω_{min} . This minimal detectable value is above the prediction for the standard inflation model assuming a tensor-to-scalar ratio $r = 0.1$, even for third generation detectors ($\Omega_{\text{min}} \simeq 10^{-13} T^{-1/2}$), meaning that the sensitivity should be improved by at least another factor of about 10 in the future to reach a level of $\Omega_{\text{min}} \sim 10^{-15}$. Notice that a pair of co-aligned and co-located detectors would permit an improvement of 50% of Ω_{min} .

Also, it is possible that a confusion background created by unresolved binary neutron stars will remain in the data at the level shown in Fig. 2. Future detections will provide constraints on the rate of such events, which will permit to estimate the level of this confusion background. An improvement of a factor of 10 in sensitivity would permit to remove all the binary sources of neutron stars and black holes.

Conclusions and discussion — In this study we have demonstrated that third generation gravitational wave detectors will have sensitivities sufficient to directly observe almost every coalescing binary black hole system in the Universe. These events are not expected to overlap in time and as such, the times containing these events can be removed from the search for a residual stochastic gravitational wave background. However a more detailed analysis is needed to assess how well one can subtract BBH signals from the data, for example using methods similar to those developed for the Big Bang Observer

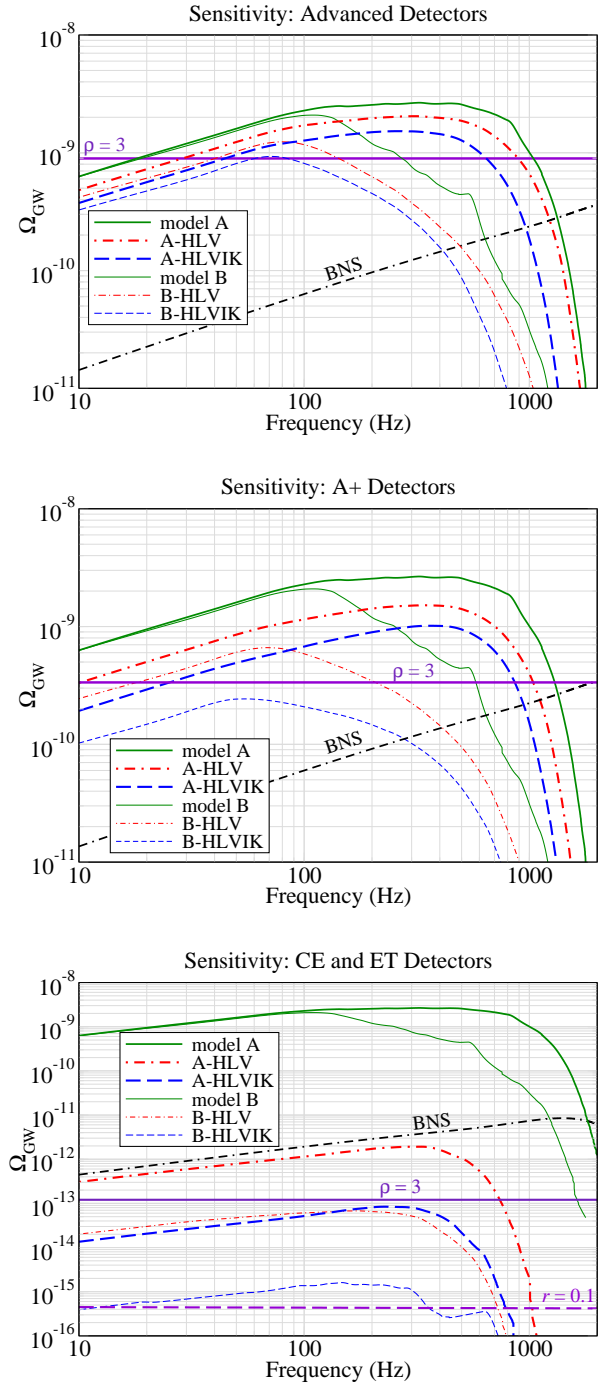


FIG. 2. Energy density spectrum Ω_{GW} in gravitational waves for the Advanced (top plot), Advanced plus (A+, middle plot) and third generation (bottom plot) sensitivities, and two different detector networks, assuming the power law (model A) and flat (model B) mass distributions for binary companions (see text for details). The cosmological background from inflation assuming a tensor-to-scalar ratio of $r = 0.1$ is shown for comparison, and confusion background from unresolved binary neutron stars, assuming an average local rate of $60 \text{ Gpc}^{-3} \text{ yr}^{-1}$ [54]. The horizontal solid line is the minimal flat spectrum that can be detected with $\rho = 3$ with a 5-detector network after five years.

[15, 16] or LISA [55]; this will be taken up in a mock data challenge to be carried out soon. With the binary black hole coalescences removed, these detectors would be sensitive to a cosmologically produced stochastic background at the level of $\Omega_{\text{GW}} \simeq 10^{-13}$, after five years of observation, comparable to the sensitivity of LISA [56]. A potential limitation to this sensitivity comes from other astrophysically produced gravitational waves, such as those from the coalescence of binary neutron stars, but there is still much uncertainty on the magnitude of this background. Observations of compact binary coalescence events in the coming years will provide the necessary information on their merger rate. The removal of BBH confusion background with third generation detectors opens up the possibility to observe PGWB.

Acknowledgments — We thank Thomas Dent, Vuk Mandic and Alan Weinstein for comments. B.S.S acknowledges the support of Science and Technologies Facilities Council grant ST/L000962/1. N.C received support from NSF grant PHY-1505373. T.R acknowledges the LIGO Visitors Program and is grateful to X.O. for useful discussions. This article has been assigned LIGO Document number P1600323.

-
- [1] L. P. Grishchuk, Soviet Journal of Experimental and Theoretical Physics **40**, 409 (1975).
 - [2] L. P. Grishchuk, Phys. Rev. D **48**, 3513 (1993), gr-qc/9304018.
 - [3] A. A. Starobinskii, ZhETF Pisma Redaktsiiu **30**, 719 (1979).
 - [4] M. Gasperini and G. Veneziano, Astroparticle Physics **1**, 317 (1993), hep-th/9211021.
 - [5] A. Buonanno, M. Maggiore, and C. Ungarelli, Phys. Rev. D **55**, 3330 (1997), gr-qc/9605072.
 - [6] J.-F. Dufaux, D. G. Figueroa, and J. García-Bellido, Phys. Rev. D **82**, 083518 (2010), arXiv:1006.0217 [astro-ph.CO].
 - [7] T. Damour and A. Vilenkin, Phys. Rev. D **71**, 063510 (2005), hep-th/0410222.
 - [8] X. Siemens, V. Mandic, and J. Creighton, Physical Review Letters **98**, 111101 (2007), astro-ph/0610920.
 - [9] S. Ölmec, V. Mandic, and X. Siemens, Phys. Rev. D **81**, 104028 (2010), arXiv:1004.0890 [astro-ph.CO].
 - [10] T. Regimbau, S. Giampanis, X. Siemens, and V. Mandic, Phys. Rev. D **85**, 066001 (2012), arXiv:1111.6638 [astro-ph.CO].
 - [11] C. Caprini, R. Durrer, and G. Servant, Phys. Rev. D **77**, 124015 (2008), arXiv:0711.2593.
 - [12] C. Caprini, R. Durrer, T. Konstandin, and G. Servant, Phys. Rev. D **79**, 083519 (2009), arXiv:0901.1661 [astro-ph.CO].
 - [13] C. Caprini, R. Durrer, and G. Servant, Journal of Cosmology and Astroparticle Physics **12**, 024 (2009), arXiv:0909.0622 [astro-ph.CO].
 - [14] T. Regimbau, Research in Astronomy and Astrophysics **11**, 369 (2011), arXiv:1101.2762 [astro-ph.CO].
 - [15] C. Cutler and J. Harms, Phys. Rev. D **73**, 042001 (2006), gr-qc/0511092.
 - [16] J. Harms, C. Mahrtdt, M. Otto, and M. Prieß, Phys. Rev. D **77**, 123010 (2008), arXiv:0803.0226 [gr-qc].
 - [17] B. P. Abbott, R. Abbott, T. D. Abbott, M. R. Abernathy, F. Acernese, K. Ackley, C. Adams, T. Adams, P. Addesso, R. X. Adhikari, and et al., Physical Review Letters **116**, 131103 (2016), arXiv:1602.03838 [gr-qc].
 - [18] G. M. Harry *et al.* (LIGO Scientific Collaboration), Classical Quantum Gravity **27**, 084006 (2010).
 - [19] J. Aasi *et al.* (LIGO Scientific Collaboration), Classical Quantum Gravity **32**, 074001 (2015).
 - [20] B. P. Abbott, R. Abbott, T. D. Abbott, M. R. Abernathy, F. Acernese, K. Ackley, C. Adams, T. Adams, P. Addesso, R. X. Adhikari, and et al., Physical Review Letters **116**, 061102 (2016), arXiv:1602.03837 [gr-qc].
 - [21] B. P. Abbott, R. Abbott, T. D. Abbott, M. R. Abernathy, F. Acernese, K. Ackley, C. Adams, T. Adams, P. Addesso, R. X. Adhikari, and et al., Physical Review Letters **116**, 241102 (2016), arXiv:1602.03840 [gr-qc].
 - [22] B. P. Abbott, R. Abbott, T. D. Abbott, M. R. Abernathy, F. Acernese, K. Ackley, C. Adams, T. Adams, P. Addesso, R. X. Adhikari, and et al., Astrophys. J. Supp. **818**, L22 (2016), arXiv:1602.03846 [astro-ph.HE].
 - [23] K. Belczynski, M. Dominik, T. Bulik, R. O’Shaughnessy, C. Fryer, and D. E. Holz, Astrophys. J. Supp. **715**, L138 (2010).
 - [24] C. L. Rodriguez, M. Morscher, B. Pattabiraman, S. Chatterjee, C.-J. Haster, and F. A. Rasio, Physical Review Letters **115**, 051101 (2015).
 - [25] The LIGO Scientific Collaboration, the Virgo Collaboration, B. P. Abbott, R. Abbott, T. D. Abbott, M. R. Abernathy, F. Acernese, K. Ackley, C. Adams, T. Adams, and et al., ArXiv e-prints (2016), arXiv:1606.04856 [gr-qc].
 - [26] The LIGO Scientific Collaboration, the Virgo Collaboration, B. P. Abbott, R. Abbott, T. D. Abbott, M. R. Abernathy, F. Acernese, K. Ackley, C. Adams, T. Adams, and et al., ArXiv e-prints (2016), arXiv:1602.03842 [astro-ph.HE].
 - [27] X.-J. Zhu, E. Howell, T. Regimbau, D. Blair, and Z.-H. Zhu, Astrophys. J. **739**, 86 (2011).
 - [28] P. A. Rosado, Phys. Rev. D **84**, 084004 (2011).
 - [29] S. Marassi, R. Schneider, G. Corvino, V. Ferrari, and S. P. Zwart, Phys. Rev. D **84**, 124037 (2011).
 - [30] C. Wu, V. Mandic, and T. Regimbau, Phys. Rev. D **85**, 104024 (2012).
 - [31] X.-J. Zhu, E. J. Howell, D. G. Blair, and Z.-H. Zhu, Monthly Notices of the Royal astronomical Society **431**, 882 (2013).
 - [32] I. Kowalska-Leszczynska, T. Regimbau, T. Bulik, M. Dominik, and K. Belczynski, Astronomy and Astrophysics **574**, A58 (2015).
 - [33] D. Meacher, M. Coughlin, S. Morris, T. Regimbau, N. Christensen, S. Kandhasamy, V. Mandic, J. D. Romano, and E. Thrane, Phys. Rev. D **92**, 063002 (2015), arXiv:1506.06744 [astro-ph.HE].
 - [34] B. P. Abbott, R. Abbott, T. D. Abbott, M. R. Abernathy, F. Acernese, K. Ackley, C. Adams, T. Adams, P. Addesso, R. X. Adhikari, and et al., Physical Review Letters **116**, 131102 (2016).
 - [35] I. Dvorkin, E. Vangioni, J. Silk, J.-P. Uzan, and K. A. Olive, ArXiv e-prints (2016), arXiv:1604.04288 [astro-ph.HE].
 - [36] K. Nakazato, Y. Niino, and N. Sago, ArXiv e-prints (2016), arXiv:1605.02146 [astro-ph.HE].

- [37] M. Punturo *et al.*, Classical Quantum Gravity **27**, 194002 (2010).
- [38] B. P. Abbott, R. Abbott, T. D. Abbott, M. R. Abernathy, K. Ackley, C. Adams, P. Addesso, R. X. Adhikari, V. B. Adya, C. Affeldt, and et al., ArXiv e-prints (2016), arXiv:1607.08697 [astro-ph.IM].
- [39] T. Regimbau, T. Dent, W. Del Pozzo, S. Giamparis, T. G. F. Li, C. Robinson, C. Van Den Broeck, D. Meacher, C. Rodriguez, B. S. Sathyaprakash, and K. Wójcik, Phys. Rev. D **86**, 122001 (2012), arXiv:1201.3563 [gr-qc].
- [40] T. Regimbau, D. Meacher, and M. Coughlin, Phys. Rev. D **89**, 084046 (2014), arXiv:1404.1134.
- [41] E. Vangioni, K. A. Olive, T. Prestegard, J. Silk, P. Petitjean, and V. Mandic, Mon. Not. Roy. Astr. Soc. **447**, 2575 (2015).
- [42] M. Kistler, H. Yuksel, and A. Hopkins, (2013), arXiv:1305.1630.
- [43] M. Trenti, R. Perna, and S. Tacchella, Astrophys. J. Supp. **773**, 22 (2013).
- [44] P. Behroozi and J. Silk, Astrophys. J. **799**, 32 (2015).
- [45] I. Dvorkin, J. Silk, E. Vangioni, P. Petitjean, and K. A. Olive, Mon. Not. Roy. Astr. Soc. **452**, L36 (2015), arXiv:1506.06761.
- [46] P. Madau and M. Dickinson, AARA **52**, 415 (2014).
- [47] K. Belczynski, S. Repetto, D. E. Holz, R. O’Shaughnessy, T. Bulik, E. Berti, C. Fryer, and M. Dominik, Astrophys. J. **819**, 108 (2016), arXiv:1510.04615 [astro-ph.HE].
- [48] P. Ajith, Phys. Rev. D **84**, 084037 (2011), arXiv:1107.1267 [gr-qc].
- [49] B. Allen and J. D. Romano, Phys. Rev. D **59**, 102001 (1999).
- [50] P. A. R. Ade *et al.* (Planck), (2015), arXiv:1502.01589 [astro-ph.CO].
- [51] J. Aasi *et al.* (LIGO Scientific Collaboration and Virgo Collaboration), arXiv **1304**, 0670 (2013).
- [52] I. B. et al., (2011), <https://dcc.ligo.org/LIGO-M1100296/public>.
- [53] Y. Aso, Y. Michimura, K. Somiya, M. Ando, O. Miyakawa, T. Sekiguchi, D. Tatsumi, and H. Yamamoto, Phys. Rev. D **88**, 043007 (2013), arXiv:1306.6747 [gr-qc].
- [54] M. Dominik, E. Berti, R. O’Shaughnessy, I. Mandel, K. Belczynski, C. Fryer, D. E. Holz, T. Bulik, and F. Pannarale, Astrophys. J. **806**, 263 (2015), arXiv:1405.7016 [astro-ph.HE].
- [55] R. Umstätter, N. Christensen, M. Hendry, R. Meyer, V. Simha, J. Veitch, S. Vigeland, and G. Woan, Phys. Rev. D **72**, 022001 (2005).
- [56] P. Binétruy, A. Bohé, C. Caprini, and J.-F. Dufaux, Journal of Cosmology and Astroparticle Physics **6**, 027 (2012), arXiv:1201.0983 [gr-qc].